Lesson 27: Antiderivatives and Indefinite Integrals

Def $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Ex If $f(x) = x^2$, then
- $F(x) = \frac{1}{3}x^3$ is an antiderivative
- $G(x) = \frac{1}{3}x^3 + 4$ is an antiderivative
- $H(x) = \underbrace{\frac{1}{3}x^3 + C}_{\text{general antiderivative}}$, C ← constant

Notation $\int \overbrace{x^2}^{\text{integrand}}\, dx = \frac{1}{3}x^3 + C$

"the antiderivative (integral) of x^2 with respect to x"

Basic Formulas

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$

$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

$\int e^x dx = e^x + C$

$\int \cos x\, dx = \sin x + C$

$\int \sin x\, dx = -\cos x + C$

etc.

$\int k f(x)\, dx = k \int f(x) dx$ (Ex: $\int 2e^x dx = 2\int e^x dx = 2e^x + C$)

$2(e^x + C) = 2e^x + 2C$ ← still a constant!

$\int f(x) + g(x)\, dx = \int f(x) dx + \int g(x) dx$ (Ex: $\int x+1\, dx = \int x\, dx + \int 1\, dx$

$= (\frac{1}{2}x^2 + C') + (x + D)$

$= \boxed{\frac{1}{2}x^2 + x + C}$)

Caution! $\int f(x)g(x)\,dx \neq \int f(x)\,dx \int g(x)\,dx$

$$\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

Ex $\int x^2\,dx = \frac{1}{3}x^3 + C$

$$\int x\,dx \int x\,dx = \left(\tfrac{1}{2}x^2 + C\right)\left(\tfrac{1}{2}x^2 + D\right)$$

Example 1 $\int \frac{x+1}{\sqrt{x}}\,dx = \int (x+1)x^{-1/2}\,dx$ $\quad (x x^{-1/2} = x^{1-1/2} = x^{1/2})$

$$= \int x^{1/2} + x^{-1/2}\,dx$$

$$= \boxed{\frac{1}{3/2}x^{3/2} + \frac{1}{1/2}x^{1/2} + C}$$

Example 2 $\int \frac{3\sin x}{\cos^2 x}\,dx = \int 3\,\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$

$$= \int 3\tan x \sec x\,dx$$

$$= 3\int \sec x \tan x\,dx$$

$$= \boxed{3\sec x + C}$$